Photoproduction of J/ψ in the forward region

James Amundson[*]

Department of Physics, University of Wisconsin, Madison, WI 53706
Current Address: Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824

Sean Fleming[†]

Department of Physics, University of Wisconsin, Madison, WI 53706

Ivan Maksymyk[‡]

Theory Group, Department of Physics, University of Texas, Austin, TX 78712
Current Address: TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada, V6T 2A3

We study the phenomenology of fixed-target elastic J/ψ photoproduction in the NRQCD factorization formalism. Our the goal is to test an essential feature of this formalism — the color-octet mechanism. We obtain an order-of-magnitude estimate for a certain linear combination of NRQCD color-octet matrix elements. Our estimate is consistent with other empirical determinations and with the v-scaling rules of NRQCD.

I. INTRODUCTION

We study the phenomenology of the NRQCD factorization formalism, which is a framework for writing inclusive rates of production and decay for quarkonium. Specifically, we consider photoproduction of J/ψ. This process probes an essential feature of the NRQCD formalism — the octet-mechanism. We approach our subject with the question: can existing photoproduction data be used to test the octet-mechanism?

II. COLOR-SINGLET APPROACHES TO PHOTOPRODUCTION OF J/ψ

Before discussing the NRQCD factorization formalism, we first discuss other theoretical frameworks for the calculation of charmonium production rates. One of the earliest computations of J/ψ production was carried out by Berger and Jones in 1981 [1]. There, the authors give an expression for the rate of production of J/ψ in γ-nucleon collisions, as calculated in the so-called "color-singlet" model. In this model, one adopts a nonrelativistic boundstate picture to describe the J/ψ [2]; calculations of J/ψ production are based upon the amplitude for the generation of a $c\bar{c}$ pair in a color-singlet $^3S_1$ configuration with small relative momentum (*i.e.* $|\mathbf{q}| \ll m_c$). The $c\bar{c}$ subprocess considered by Berger and Jones is shown in Fig. 1. In it, a gluon from the nucleon "fuses" with the photon to form a hadronizing $c\bar{c}$ pair that recoils against a gluon jet. These diagrams represent the leading color-singlet contribution to

$$\gamma + N \rightarrow J/\psi + X . \tag{1}$$

The color-singlet model formulation of J/ψ photoproduction has been studied extensively [3]. These analyses hinge on the experimental kinematic parameter

$$z \equiv \frac{P_{J/\psi} \cdot P_N}{P_\gamma \cdot P_N}, \tag{2}$$

where $P_{J/\psi}$ is the J/ψ four-momentum, P_N is the four-momentum of the initial state nucleon, and P_γ is the initial state photon four-momentum. (Note that in the lab frame of fixed target experiments, one has $z = E_\psi/E_\gamma$.) It is found that once next-to-leading order QCD corrections to the Berger-Jones results are taken into account, the color-singlet model can adequately explain the experimental data for the kinematic regime $p_T \geq 1\text{GeV}$ and $z \leq 0.8$, where p_T is the transverse momentum of the J/ψ in the center-of-mass frame.

---

[*]Email: amundson@pa.msu.edu

[†]Email: fleming@phenxs.physics.wisc.edu

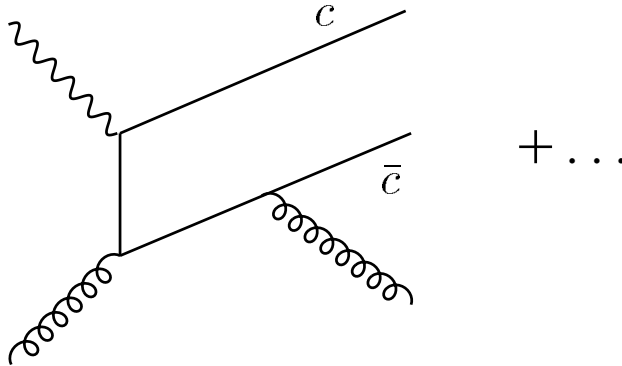[‡]Email: maksymyk@alph02.triumf.ca

FIG. 1. Leading order diagrams for the photoproduction of a $c\bar{c}$ in a color-singlet, $^3S_1$ state.

However, the color-singlet model alone cannot explain the total J/ψ photoproduction cross section. It severely underestimates the rate of production in the region $z \geq 0.9$ [4] [5], which (importantly) is the region containing nearly all of the events. In production of J/ψ in this high-z range, the "X" in Eq. 1 represents either an elastically scattered nucleon, or, a nucleon (or resonance thereof) and light hadrons that have very low energy in the rest frame of the final state nucleon. Such events, sometimes called "diffractive scattering," have been modeled in Refs. [6] and [7]. The essential features of the diffractive mechanism are the creation of a heavy quark pair from the incoming photon and the exchange of some color-singlet gluon combination between the $c\bar{c}$ pair and the proton. (Diffractive processes can in general entail the creation of a heavy quark pair from an incoming gluon also.) In the analysis of Refs. [6] and [7], the amplitude for the completely exclusive process $\gamma p \to J/\psi\, p$ is factored into three pieces: the process $\gamma \to c\bar{c}$, the scattering of the $c\bar{c}$ system on the proton via (colorless) two (or multiple) gluon exchange, and the formation of the J/ψ from the outgoing $c\bar{c}$ pair. The exchanged gluons are taken to have very low transverse momentum. This mechanism is considered in Ref. [7] to serve as a probe of the gluon density in the proton, the cross-section being proportional to $[x f_{g/N}(x)]^2$:

$$d\sigma(\gamma p \to J/\psi p)/dt \propto \left[x f_{g/N}(x)\right]^2 \tag{3}$$

where x is a dimensionless ratio of kinematic variables. Indeed there is remarkably good agreement between the formula of Refs. [6] and [7] and high-z fixed target and HERA data (see Fig. 2 of Ref. [7]).

The photoproduction cross section can be described at low z by the color-singlet model, and at high z by the approach of Refs. [6] and [7]. It is interesting however to consider the possibility that the total J/ψ photoproduction cross section can be parametrized within a single coherent theoretical framework. Such a framework is provided by the nonrelativistic QCD (NRQCD) factorization formalism of Bodwin, Braaten, and Lepage [8]. This formalism is intended only for the parameterization of inclusive production of quarkonium. Here, "inclusive" means that a perturbative partonic process produces the heavy quark pair and (possibly) some other specified final state partons; the heavy quark pair hadronizes into a specific quarkonium boundstate but no mention is made in the formalism of the fate of the other partons regarding the baryons into which they hadronize; all possibilities are *included*; moreover, a large number of light quanta (generally pions) are expected to accompany this *inclusive* production.

At the outset, the choice between the diffractive scattering formalism and the NRQCD factorization formalism hinges on the questions that one asks, that is whether one desires to model exclusive or inclusive events. We will see below however that the line between these two cases is blurred by the fact that in the set of all data, a large portion of the J/ψ photoproduction events have a certain exclusive character — that is, the final state contains very few of the extra quanta (such as pions) that one would expect in inclusive production.

III. NRQCD FACTORIZATION FORMALISM

According to the NRQCD factorization formalism, the inclusive cross section for quarkonium production is expressed as a sum of products known as a "factorization formula." For the photoproduction of J/ψ, the factorization formula is

2

$$\sigma(\gamma + N \to J/\psi + X) = \sum_n \frac{F(n)}{m_c^{d_n-4}} \langle 0|\mathcal{O}^{J/\psi}(n)0| \rangle \,. \tag{4}$$

In the above expression, the index n labels the initial color and angular-momentum quantum numbers of the $c\bar{c}$ pair produced by short-distance physics. The short-distance coefficients $F(n)$ contain only effects of distance scales of order $1/m_c$ (where m_c is the charm quark mass) or smaller; they can be calculated, using Feynman diagrams, as a perturbative expansion in $\alpha_s(m_c)$. Effects of longer distances, including effects related to the hadronization of the $c\bar{c}$ pair into the J/ψ boundstate, are parametrized by the NRQCD matrix elements $\langle 0|\mathcal{O}^{J/\psi}(n)|0 \rangle$; their relative importance can be determined using the NRQCD v-scaling rules [9], where v is the typical relative velocity of the c and $\bar{c}$ in the J/ψ boundstate. d_n is an integer associated with the energy dimension of the operator $\mathcal{O}^{J/\psi}(n)$.

The NRQCD formalism has several features that make it highly compelling. Firstly, it is rigorous in that it is based upon an effective Lagrangian for heavy quarks derived directly from full QCD. Secondly, it goes beyond the color-singlet model in that it takes into consideration processes in which a heavy quark pair is produced initially in a *color-octet* state and attains color-neutrality and hadronizes via the emission and absorption of soft gluons; the rates due to such color-octet channels are rigorously and systematically parametrized in terms of NRQCD matrix elements. Finally, the NRQCD factorization formalism provides a solution to the problem of the infrared divergences that arise in naive color-singlet-model calculations of P-wave quarkonia decay and production [10]; in the NRQCD framework, these divergences are absorbed into appropriate (color-octet) matrix elements.

The factorization hypothesis expressed in Eq. 4 is considered to be most valid when applied to differential cross sections with p_T much greater then the hadronic scale 1 GeV [29]. However, it is not yet clearly understood in the NRQCD theoretical community whether *total* cross sections can be expressed in the factorized form. In this paper we will assume that they can be. If our assumption is valid then one expects relative corrections from higher-twist operators to Eq. 4 to be [11]

$$O\left((1 \text{ GeV})^2/m_c^2\right), \tag{5}$$

with m_c in GeV.

IV. NRQCD PREDICTION FOR PHOTOPRODUCTION

Let us now turn to a discussion of the NRQCD prediction for the total cross section for photoproduction of J/ψ. To write down the leading pieces of the NRQCD prediction, we must first determine which are the most important terms in the factorization formula. The numerical sizes of the NRQCD matrix elements can be estimated by determining how they scale with v, which, as stated earlier, is the typical velocity of the heavy quarks in the quarkonium boundstate. ($v_c^2 \simeq 0.25$.) Combining v-scaling estimates of $\langle 0|\mathcal{O}^{J/\psi}(n)|0 \rangle$ with the α_s-scaling of the $F(n)$, it is possible to determine the relative importance of the terms in the factorization formula, Eq. 4, in regard to the double expansion in v and α_s. One finds that the leading contributions to the total photoproduction cross section come from the production of a $c\bar{c}$ pair in a color-singlet $^3S_1$ state (the Feynman diagrams are shown in Fig. 1), and from the production of a $c\bar{c}$ pair in a color-octet $^1S_0$, $^3P_0$, or $^3P_2$ state (the Feynman diagrams are shown in Fig. 2). The leading color-singlet contribution to the rate is proportional to $\alpha_s^3 v^3$, while the leading color-octet contributions to the rate are proportional to $\alpha_s^2 v^7$. Since $\alpha_s(m_c) \sim v^2$, we expect the two effects to contribute roughly with the same size to the total cross-section.

Note that in the case of J/ψ production where the $c\bar{c}$ pair is produced initially in a color-singlet $^3S_1$ state (as in Fig. 1), there must necessarily be a final state hard gluon, for color conservation. If this hard gluon is of relatively low (high) energy, then the J/ψ is produced in the high-z (low-z) region. For the color-singlet contribution, we thus see that there exists some continuous z-distribution. On the other hand, as to the case (in Fig. 2) in which the $c\bar{c}$ pair is initially produced in a color-octet $^1S_0$, $^3P_0$, or $^3P_2$ state, there is no hard final state gluon at leading order; the color-octet mechanism contributes therefore only to the high-z region at leading order. The naive picture is that the z-distribution from this sort of contribution is an idealized Dirac function at $z = 1$. We will refer to the color-octet process in Fig. 2 as "forward-octet."

We now present the color-singlet J/ψ photoproduction rate, the underlying process of which is shown in Fig. 1. The NRQCD result for the color-singlet contribution is simply proportional to the color-singlet model expression [1] [3]

$$\frac{d\sigma_{\text{CS}}}{dt} = \frac{64\pi e_c^2 \alpha_{em} \alpha_s(t)^2 m_c |R_s(0)|^2}{3s^2} \frac{s^2(s-4m_c^2)^2 + t^2(t-4m_c^2)^2 + u^2(u-4m_c^2)^2}{(s-4m_c^2)^2(t-4m_c^2)^2(u-4m_c^2)^2} \tag{6}$$

where $\alpha_s(t)$ is the strong coupling constant evaluated at the typical scale of the interaction, t, where α_{em} is the electromagnetic coupling constant, where e_c^2 is the fractional charm quark charge, and where s, t, and u are the usual
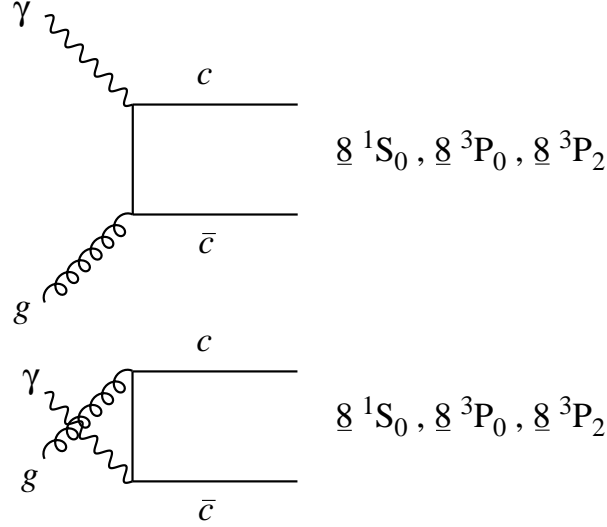
FIG. 2. Leading order diagrams for the photoproduction of a $c\bar{c}$ in a color-octet state with angular momentum configuration $^1S_0$, $^3P_0$, and $^3P_2$. These diagrams are the underlying subprocess for the forward octet contributions to J/ψ photoproduction.

Lorentz invariant Mandelstam variables. Here $R_s(0)$ is the radial color-singlet wavefunction evaluated at $\mathbf{x} = 0$. It is related to the color-singlet $^3S_1$ production matrix element through

$$\langle 0|\mathcal{O}_1^{J/\psi}(^3S_1)|0\rangle = \frac{9}{2\pi}|R_s(0)|^2\left(1 + O(v^2)\right) . \tag{7}$$

We next present the leading color-octet ("forward octet") photoproduction rate, the underlying process of which is shown in Fig. 2. The factorization formula for the forward-octet contribution can be written as follows:

$$\sigma(\gamma g \to J/\psi + X)_{\text{FO}} = \frac{F(8,{}^1S_0)}{m_c^2}\langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$$
$$+ \frac{F(8,{}^3P_0)}{m_c^4}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle + \frac{F(8,{}^3P_2)}{m_c^4}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_2)|0\rangle . \tag{8}$$

One can lift (with the help of a suitable color-factor replacement), the short-distance coefficients $F(8,{}^1S_0)$ and $F(8,{}^3P_J)$, from the results for the process $gg \to J/\psi$ given in Ref. [12]. One obtains

$$F(8,{}^1S_0) = \frac{\pi^3 e_c^2 \alpha_s(2m_c)\alpha_{em}}{m_c} \delta(4m_c^2 - s)$$

$$F(8,{}^3P_0) = \frac{3\pi^3 e_c^2 \alpha_s(2m_c)\alpha_{em}}{m_c} \delta(4m_c^2 - s)$$

$$F_8(8,{}^3P_2) = \frac{4}{5}\frac{\pi^3 e_c^2 \alpha_s(2m_c)\alpha_{em}}{m_c} \delta(4m_c^2 - s) . \tag{9}$$

Inserting these into the factorization formula in Eq. 8, we obtain the subprocess cross-section

$$\sigma(\gamma g \to J/\psi + X)_{\text{FO}} = \frac{\pi^3 e_c^2 \alpha_s(2m_c)\alpha_{em}}{m_c^3} \delta(4m_c^2 - s)\,\Theta , \tag{10}$$

where Θ is given by

$$\Theta \equiv \langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle + \frac{7}{m_c^2}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle . \tag{11}$$

The above expression is derived with the help of the relation [8]

4

$$\langle 0|\mathcal{O}_8^{J/\psi}(^3P_J)|0\rangle = (2J+1)\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle \left(1 + O(v^2)\right). \tag{12}$$

We next convolute the subprocess cross sections given in Eq. 6 (color-singlet) and Eq. 10 (forward-octet) with the gluon distribution function to obtain the leading NRQCD factorization formalism prediction for the *total* photoproduction cross section:

$$\sigma(\gamma N \to J/\psi + X)$$

$$= \int dx \, f_{g/N}(x) \left(\sigma_{\mathrm{CS}}(\gamma g \to J/\psi + X) + \sigma_{\mathrm{FO}}(\gamma g \to J/\psi + X) \right)$$

$$= \frac{\pi^2 \alpha e_c^2}{m_c^3} \int dx \, f_{g/N}(x) \left(\pi \alpha_s(2m_c) \, \Theta \, \delta\left(4m_c^2 - \hat{s}\right) \; + \right.$$

$$\left. \int d\hat{t} \frac{128\alpha_s^2(\hat{t})m_c^4 \langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}{27\hat{s}^2} \, \frac{\hat{s}^2(\hat{s}-4m_c^2)^2 + \hat{t}^2(\hat{t}-4m_c^2)^2 + \hat{u}^2(\hat{u}-4m_c^2)^2}{(\hat{s}-4m_c^2)^2(\hat{t}-4m_c^2)^2(\hat{u}-4m_c^2)^2} \right), \tag{13}$$

where $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the usual Mandelstam variables for the subprocess cross section, and $f_{g/N}(x)$ is the gluon structure function.

V. COMPARISON TO EXPERIMENT: CAVEATS

The leading NRQCD factorization formula for the total photoproduction cross section (Eq. 13) depends on the two phenomenological quantities: $\langle 0|\mathcal{O}_1^{J/\psi}(^3S_1)|0\rangle$, and $\Theta \equiv \langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle + \frac{7}{m_c^2}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$. The first parameter (the color-singlet matrix element), is well determined from measurements of the decay rate of J/ψ to two leptons [13]. As to the second parameter (the forward-octet combination Θ), the matrix elements contained therein are poorly constrained thus far, and so a testable prediction of the photoproduction cross section is not yet possible.

Given this state of affairs, we propose to use the factorization formula in Eq. 13 to obtain an estimate of the value of Θ via a comparison with fixed target photoproduction data. We find that a determination of Θ is complicated by the manner in which J/ψ photoproduction data is presented in the experimental literature. Typically, measurements of the *total inclusive* cross section are not made; thus, our Eq. 13 — a total inclusive cross section — cannot be associated in an unqualified straightforward manner with some published empirical result. We must contend with the fact that data is taken in either the *inelastic* or *elastic* regime. The *inelastic* regime is generally considered to be the region where z is below 0.8 or 0.9. On the other hand, the conventional definition of the *elastic* regime is not at all well established; it is in general considered to be the region near $z = 1$, with the data consisting of those events in which is detected a muon pair with invariant mass M_ψ along with hadrons having up to about 5 GeV of additional energy. To test the forward-octet feature of NRQCD (which is our goal), it would seem at the outset that a good strategy would be to select some range of z (from a lower bound around 0.8 up to unity) for which to calculate the total (color-singlet plus color-octet) theoretical prediction, and to compare that result to the elastic data. However the contribution of the color-singlet term is quite negligible for high z, and is actually found to be nearly an order of magnitude below the data [4] [5]. The unimportance of the color-singlet contribution $d\sigma_{\mathrm{CS}}/dz$ at high z and the vagueness surrounding the correct lower bound of z to use when integrating over z in order to compare to elastic data suggest that we might as well neglect the color-singlet contribution altogether. Why not simply go ahead and compare the elastic data to the forward-octet contribution? Before we proceed, some reservations must first be discussed.

A major reservation concerns the issue of "z-smearing." Although we ultimately do not take this stance, it is arguable that one actually does *not* expect the forward-octet piece to contribute significantly to photoproduction events for which z is nearly unity. Although the subprocesses described in Fig. 2 appear, at a first glance, to create J/ψ particles with total energy close to that of the incoming photon (*i.e.* with $z = (E_{J/\psi})/(E_\gamma) = 1$), this is not exactly the case since the $c\bar{c}$ system must emit or absorb gluons in order to make the transition to a color-singlet state and hadronize into a J/ψ boundstate. Such gluons typically have energy and momenta of order $m_c v^2$ in the rest-frame of the $c\bar{c}$ system [8]. The presence of these gluons implies that the ratio of the energy of the final charmonium particle to that of the initially produced heavy quark pair is not exactly unity but, rather, is expected to be $<z>=1-O(v^2)$. The z-distribution is smeared away from the idealization of the Dirac delta $\delta(1-z)$. Similar reasoning has been given in [14]. In fact, even in quarkonium production events involving color-singlet channels, one still might expect such a disparity between the energy of the initially produced heavy quark color-singlet pair and the final boundstate; the heavy quarks emit pions and other quanta so as to adjust their energy-momentum to that of heavy quarks in the boundstate.

The above arguments concerning z-smearing seem reasonable, but experimental information challenges them severely. In fact, quite contrary to what one might expect, it turns out that the great bulk of photoproduction events actually lie in bins above $z = 0.95$. For example, of the approximately 850 J/ψ photoproduction events collected by the H1 experiment, 700 lie in the bin above $z = 0.95$ [5]. This includes both the events in which the proton is reconstructed, and those in which the proton is dissociated. So, assuming that these events are due to the NRQCD forward octet mechanism, we must conclude that the smearing of the z-distribution is not a big effect. [1]

There is yet another reason for which, at the outset, one might have reservations concerning the fitting of Eq. 13 to high-z fixed target elastic data: the question of the inherently inclusive nature of the NRQCD formalism versus the possible lack of complete inclusiveness in the data. However, as it happens, the fixed target data *do* appear to be sufficiently inclusive for our purposes. This is because the H1 data [5], as was mentioned just above, show that most photoproduction events are associated with $z > 0.95$. If we can count on all high energy ($\sqrt{s} \gg M_\psi$) J/ψ photoproduction to have this feature, then the elastic fixed target data that we are using should surely include the great majority of events, and can be considered inclusive.

The above arguments support the decision to simply associate the high-z fixed target data with the NRQCD forward-octet contribution.

VI. COMPARISON TO EXPERIMENT: DETERMINATION OF Θ

We now proceed to use experimental data from the E687, NA14, E401, NMC and E516 experiments to evaluate Θ. We will then show that the value we determine in this way is consistent with HERA (H1 [5] and ZEUS [15]) data.

In Fig. 3 we display our fit of the forward-octet piece in Eq. 13 to elastic cross-section measurements from the fixed target Fermilab experiments E687 [16], NA14 [17], E401 [18], NMC [19], and E516 [20]. In the same figure we also show data from the SLAC experiment [21]; we do not however include this data in our fit since it falls in the low energy ($\sqrt{s} \sim M_\psi$) regime. As indicated in Fig. 2, we have performed our analysis using CTEQ [22], MRS [23], and GRV [24] structure functions. We have taken $m_c = 1.5$ GeV and $\alpha_s(2m_c) = 0.26$. With this choice of parameters we find

$$\Theta = 0.02 \, \text{GeV}^3 \ . \tag{14}$$

Note that there are large theoretical errors associated with this determination of Θ. These will be discussed in the next section.

Having estimated Θ from fixed target Fermilab data, it is interesting to see whether we can use this result to make a successful theoretical prediction for the higher energy elastic data from HERA. The prediction is shown in Fig. 4, where it is seen that our extrapolated theoretical curve (generated with $\Theta = 0.02 \text{GeV}^3$) is indeed consistent with HERA data. On this plot we show data from the experiments H1 [5] and ZEUS [15]. (Also shown on this plot are measurements from the Fermilab experiments E401 [18] and E516 [20], which are a class of data that the HERA collaboration includes on their own plots for comparison purposes. Note that the E401 and E516 data appearing in this figure are not the same as those appearing in Fig. 3.)

VII. THEORETICAL UNCERTAINTIES

Here we discuss the sources of theoretical error bearing upon our determination of Θ. We have already stated that one expects an error do to the neglect of "higher-twist" operators of $O\left((1 \text{ GeV})^2/m_c^2\right)$. Apart from this limitation, there are other important sources of theoretical uncertainty. These are associated with α_s corrections, with relativistic v^2 corrections, and with the value of the parameter m_c. We now address each of these issues.

As to α_s and relativistic corrections, to determine their size, one really must calculate them. When one is not in possession of such results, the order-of-magnitude of the corrections can nevertheless be estimated by examining similar processes where next-to-leading order corrections have been computed. In this matter we are fortunate, because there exist calculations of the α_s corrections [3] and v^2 corrections [25] to the *color-singlet* term, Eq. 6. The

---

[1]It must be mentioned in passing that in the exclusive diffractive scattering, as well, one would expect $z-$-smearing to occur. The c and $\overline{c}$, which emerge from the diffractive scattering in a color-singlet state, would have to readjust their momenta to that of quarks in the quarkonium boundstate, and this necessarily entails the emission of soft quanta of momentum of the order of mv^2.
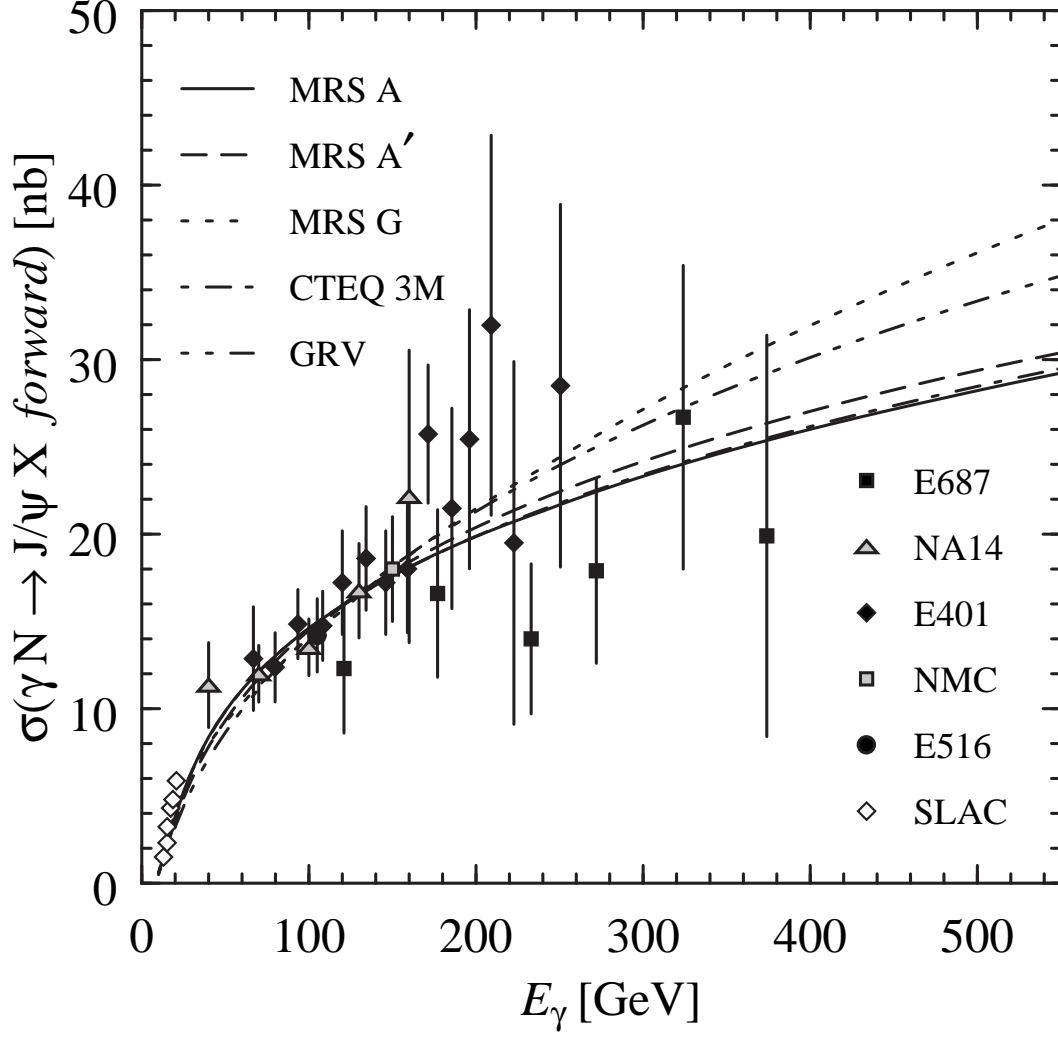
FIG. 3. Fit to fixed-target experiments. Shown are data points for the elastic photoproduction cross section measured at fixed-target experiments. Also shown are the theoretical curves evaluated numerically using five different gluon distribution functions. In the curve Θ is adjusted to best fit the data.
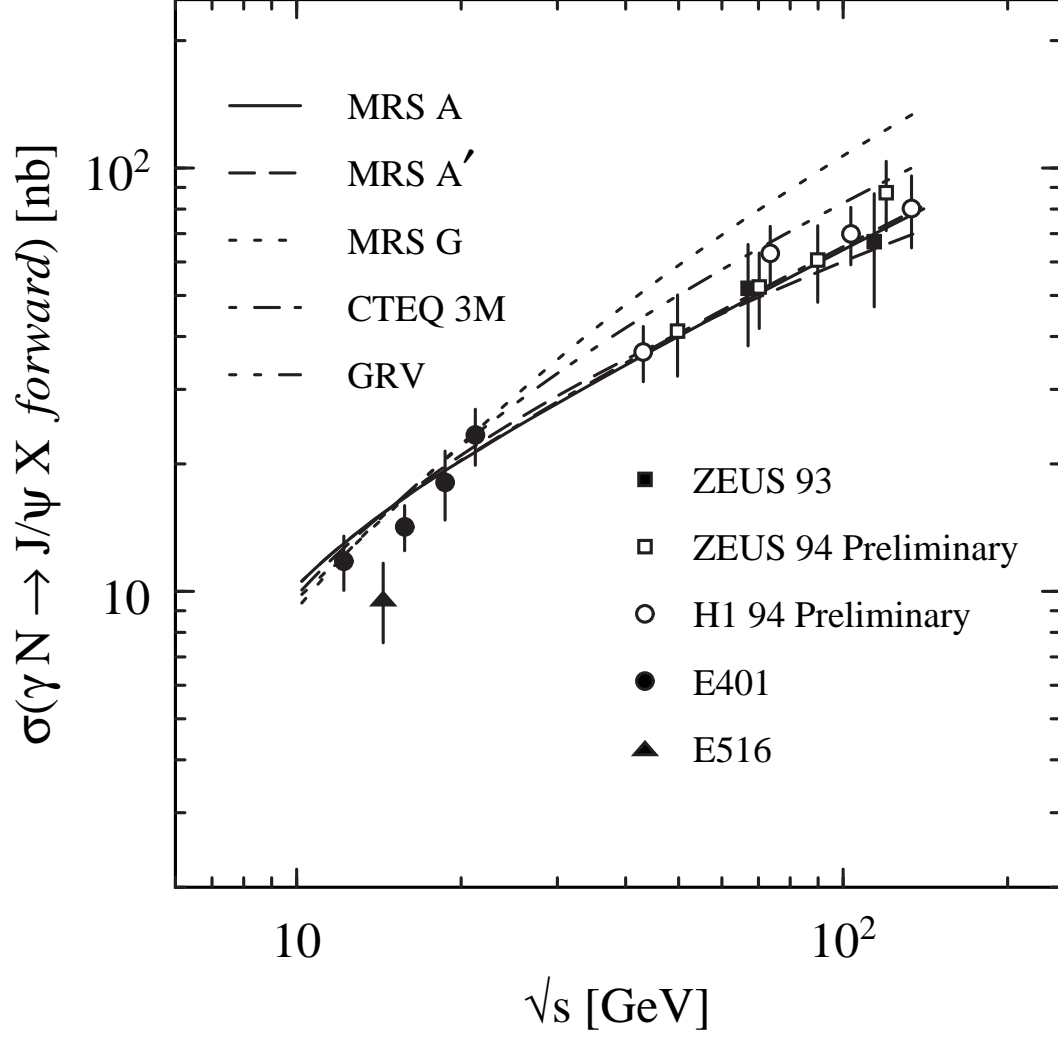
FIG. 4. Comparison of NRQCD forward octet theoretical prediction to HERA data using the value of Θ determined from fixed target experiments. Shown are data points for the elastic scattering cross section measured at various HERA and Fermilab experiments.

extrapolations of these results to a color-octet production process must be considered no better than educated guesses; we seek only order-of-magnitude guidance. According to Ref. [3], the α_s corrections to the color-singlet piece increase the theoretical result for that part of the rate by roughly a factor of two; we expect α_s corrections of similar size therefore for the forward-octet terms. Concerning relativistic corrections on the other hand, the results of Ref. [25] show that v^2 corrections to the color-singlet part of the rate contribute roughly an additional 50%, and so we expect the same for the forward-octet part of the rate.

As to the theoretical error due to uncertainty regarding the value of the parameter m_c, we estimate this by varying m_c over the range 1.3 GeV to 1.7 GeV. We find that over this range, the cross section varies by a factor of 4.

Clearly, in light of the large size of the various theoretical uncertainties, the value we determine for Θ can only be regarded as an order-of-magnitude estimate.

The uncertainty due to the gluon structure function is negligible in our analysis. We observe that different standard parameterizations of the gluon structure function yield results that vary little compared to the relatively large errors discussed above. Therefore, it is not possible to make any conclusions about the gluon content of the nucleon in the present context.

VIII. COMPARISON TO OTHER WORK

We now confront other measurements of the NRQCD J/ψ production matrix elements with our measurement of Θ.

Beneke and Rothstein [14] calculate the leading terms in the NRQCD factorization formula for $\sigma(\pi N \to J/\psi + X)$. This hadroproduction reaction involves — among other processes — the forward octet process in Fig. 2, but with two incoming gluons instead of the photon and gluon as in photoproduction. As a consequence, $\langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$ and $\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$ appear in hadroproduction rates in the exact same linear combination (Θ) as they do in photoproduction. Comparing their factorization formula to data, Beneke and Rothstein obtain $\Theta = 0.03$ GeV^3. This is consistent with our result, given the large theoretical uncertainties.

A different linear combination of $\langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$ and $\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$ is determined from an analysis of J/ψ production at the Tevatron [26]. A fit of theory to CDF data for J/ψ produced at moderate p_T gives

$$\langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle + \frac{3.5}{m_c^2}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle = 4.38 \pm 1.15^{+1.52}_{-0.74} \times 10^{-2} \text{ GeV}^3 \ . \tag{15}$$

Here we have quoted the fit done by Beneke and Krämer using the CTEQ4L structure function. Again, within the theoretical uncertainty, this result is consistent with ours.

It is interesting to consider the possibility that our value for Θ (determined from photoproduction) is accurate, and that the measurement given in Eq. 15 (determined at CDF) is also accurate. Upon solving Eqs. 14 and 15, one finds that $\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$ is negative. Though this may at first appear to be inconsistent and even meaningless, it is actually not unreasonable, if one considers that the matrix elements determined here are renormalized matrix elements [27].

The *elastic* photoproduction calculation carried out in the present paper has also been carried out by Cacciari and Krämer [4]. The expression they obtain for the forward-octet terms is in agreement with our Eq. 10. However, in their analysis, these authors arrive at conclusions different from ours. Cacciari and Krämer assume that the production matrix elements must be positive. They use the result of the analysis of Cho and Leibovich [26] (which is similar to the Beneke and Kramer result, given here in Eq. 15), to estimate the order of magnitude of $\langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle$ and of $\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$. They find that, under these assumptions, the forward-octet term in the theoretical prediction for the low-p_T high-z cross section is one order of magnitude greater than experiment. Their conclusion is that the NRQCD factorization formalism might be in conflict with the data. This potential dilemma is alleviated, though, by the fact that, as is pointed out above, not all the matrix elements need be positive.

Cacciari and Krämer also calculate the color-octet contribution to the *inelastic* (moderate z) photoproduction cross section $d\sigma/dz$. (We have not computed the color-octet contribution to this regime of the differential cross-section.) Cacciari and Krämer conclude once more that the NRQCD factorization formalism appears at variance with data. In this instance, not only is the theoretical prediction several times greater than the data, but, this time, the result of the computation of the cross-section (here, the inelastic color-octet cross-section) *cannot* be affected appreciably by the fact that $\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$ can be negative [28]. Nevertheless, it is clear from the long list of sources of theoretical uncertainty that no strong conclusions can be drawn from this discrepancy regarding the correctness of the NRQCD factorization formalism.

IX. CONCLUSIONS

We have calculated the leading order (in coupling constants and v) color-singlet and color-octet contributions to the NRQCD factorization formula for the photoproduction of J/ψ. At this order the total cross section depends on the two phenomenological parameters: $\langle 0|\mathcal{O}_1^{J/\psi}(^3S_1)|0\rangle$, and $\Theta \equiv \langle 0|\mathcal{O}_8^{J/\psi}(^1S_0)|0\rangle + \frac{7}{m_c^2}\langle 0|\mathcal{O}_8^{J/\psi}(^3P_0)|0\rangle$. Though the color-singlet matrix element is well determined, the color-octet matrix elements contained in the forward-octet combination Θ are poorly constrained. Therefore a testable prediction of the photoproduction cross section cannot be generated.

Be that as it may, it is nonetheless possible to use fixed target elastic photoproduction data to obtain a rough estimate of Θ. There are some reservations to this approach, though. Firstly, there is the issue of the "z-smearing" of the forward-octet contribution. One might be concerned that the forward-octet contribution is centered around $<z> = 1 - O(v^2)$, not $z = 1$. Secondly, there is the issue of inclusiveness. NRQCD factorization formulas apply to inclusive production, and it would seem that the region $z > 0.95$ is not inclusive enough. However both of these arguments are challenged by experimental data collected by the H1 collaboration, which shows that of 850 J/ψ photoproduction events 700 are contained in the region $z > 0.95$.

Proceeding with a fit of the forward-octet contribution to fixed target J/ψ elastic photoproduction data we determine

$$\Theta = 0.02 \text{ GeV}^3. \tag{16}$$

Given the large theoretical uncertainties associated with this calculation, it is best to regard this determination as an order of magnitude estimate. We also show that, using the value of Θ quoted above, we can explain the higher energy HERA data. Furthermore the value we determine for Θ is consistent with other determinations of the color-octet matrix elements from hadronic J/ψ production at fixed target and collider experiments. Moreover, our value of Θ is consistent with the v-scaling rules of NRQCD.

It must be appreciated that our NRQCD forward-octet result for the photoproduction rate in linear is the gluon structure function of the proton (see Eq. 13) while the diffractive rate calculated in Ref. [6] and [7] is quadratic in the gluon structure function (see Eq. 3). Thus, the two methods are essentially irreconcilable. However, one observes that both approaches are capable of convincingly describing data. The extent to which one might "prefer" NRQCD over diffractive scattering as the correct explanation hinges on whether one accepts the hypothesis that, when calculating the production rate for heavy quarks, to avoid double-counting one ought not include certain additional mechanisms that enhance cross-sections such as a diffractive scattering [29].

We have failed to resolve this issue. Perhaps a valuable diagnostic test to probe the photoproduction mechanism would be to compute various polarization rates in both formalisms, for confrontation with experiment. This includes polarization of the incoming photon, as well as of the final J/ψ. Also, a valuable diagnostic would be to compare data to various differential cross-sections calculated in both formalisms.

[1] E.L. Berger and D. Jones, Phys. Rev. **D23** (1981) 7.

[2] See E. Braaten, S. Fleming, and T.C. Yuan, Ann. Rev. Nucl. Part. Sci. **46** (1996) 197, and references therein.

[3] M. Krämer, Nucl. Phys. **B459** (1996) 3.

[4] M. Cacciari and M. Krämer Phys. Rev. Lett. **76** (1996) 4128.

[5] S. Aid *et al.*, Nucl. Phys. **B472** (1996) 3.

[6] S.J. Brodsky, L. Frankfurt, J.F. Gunion, A.H. Mueller, and M. Strikman, Phys. Rev. **D50** (1994) 3134.

[7] M.G. Ryskin, R.G. Roberts, A.D. Martin, and E.M. Levin, preprint DTP-95-96 (hep-ph/9511228).

[8] G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. **D51** (1995) 1125.

[9] G.P. Lepage, L. Magnea, C Nahkleh, U. Magnea, and K. Hornbostel, Phys. Rev. **D46** (1992) 4052.

[10] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. **D46** (1992) 1914.

[11] Geoff Bodwin, private communication.

[12] S. Fleming and I. Maksymyk, Phys. Rev. **D54** (1996) 3608.

[13] G.T. Bodwin, D.K. Sinclair, and S. Kim, Phys.Rev.Lett. **77** (1996) 2376.

[14] M. Beneke and I.Z. Rothstein, Phys.Rev. **D54** (1996) 2005; erratum-ibid. **D54** (1996) 7082.

[15] M. Derrick *et al.*, ZEUS Coll., Phys. Lett. **B350** (1995) 120; J.N. Lim for the ZEUS Coll., presented at the 27th International Conference on High Energy Physics, Glasgow, 20-27 July 1994.

[16] P.L. Frabetti, *et al.*, Phys. Lett. **B316** (1993) 197.

[17] R. Barate, *et al.*, Z. Phys. C **33** (1987) 505.

[18] M. Binkley, *et al.*, Phys. Rev. Lett. **48** (1982) 73.

[19] M. Arneodo, *et al.*, Phys. Lett. **B332** (1994) 195.

[20] B.H. Denby, *et al.*, Phys. Rev. Lett. **52** (1984) 795.

[21] U. Camerini, *et al.*, Phys. Rev. Lett. **35** (1975) 483.

[22] H.L. Lai, *et al.*, Phys. Rev. **D51** (1995) 4763.

[23] A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Rev. **D50** (1994) 6734; Phys. Rev. **D51** (1995) 4756; Phys. Lett. **B354** (1995) 155.

[24] M. Gluck, E. Reya and A. Vogt, Z. Phys. **C67** (1995) 433.

[25] W.-Y. Keung and I.J. Muzinich, Phys. Rev. **D27**, (1983) 1518.

[26] P. Cho and A.K. Leibovich, Phys.Rev. **D53** (1996) 150; Phys.Rev. **D53** (1996) 6203; M. Beneke and M. Krämer, CERN preprint CERN-TH-96-310 (hep-ph/9611218).

[27] S. Fleming, O. Hernández, I. Maksymyk, and H. Nadeau, U. Wisconsin preprint MADPH-96-953 (hep-ph/9608413); S. Fleming, Presented at Quarkonium Physics Workshop: Experiment Confronts Theory, Chicago, IL, 13-15 Jun 1996, U. Wisconsin preprint MADPH-96-966 (hep-ph/9610372).

[28] M. Krämer, Presented at Quarkonium Physics Workshop: Experiment Confronts Theory, Chicago, IL, 13-15 Jun 1996, DESY preprint DESY-96-201 (hep-ph/9609416).

[29] J.C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B263** (1986) 37.